Flow–induced vibrations of two mechanically coupled pivoted circular cylinders: characteristics of vibration

Hamid Arionfard\textsuperscript{a}, Yoshiki Nishi\textsuperscript{a,∗}

\textsuperscript{a}Department of Systems Design for Ocean-Space, Faculty of Engineering, Yokohama National University, 79-5 Tokiwadai, Hodogaya, Yokohama, Japan

Abstract

This paper presents the results of an experimental investigation on the vibration mechanisms of two connected circular cylinders that are free to rotate around a pivot in different arrangements including both cylinders on the downstream, both on the upstream and a cylinder on each side of the pivot point. The Reynolds number is varied during the test to find the maximum possible displacement amplitude for each configuration. Four main mechanisms of vibration are identified. The cylinders experience galloping if both located on the upstream of the pivot and the gap between them is zero. Vortex excitation (VE) is observed in two configurations and referred to as VE\textsubscript{fSt} and VE\textsubscript{fN}. VE\textsubscript{fSt} occurs when both cylinders are located on the downstream of the pivot while the gap is zero. The frequency responses lock into the Strouhal frequency in this case. VE\textsubscript{fN} occurs when the center of gravity (\(c_g\)) is on the pivot and the gap ratio (\(G = \text{gap/cylinder diameter}\)) between the cylinders is \(G > 3.9\). In this case, the frequency response locks into the natural frequency of the system. If one cylinder is located on the center of the rotation and the other cylinder is on the downstream, wake–induced vibration (WIV) takes place. While for \(G < 1.4\) the response is a typical wake galloping, for \(G > 1.4\) two vibration modes are recognizable as 'combined vortex resonance and galloping'. For all configurations with \(G > 0\), gap–switching–induced vibration (GSIV) is observed especially for \(1.9 < G < 2.4\). However, GSIV is the dominant mechanism of vibration if \(c_g\) is on the pivot point. In cases where \(c_g\) is not close to the pivot, the drag force may enhance the vibration if the Reynolds number is not large enough to suppress the motion.

Keywords: FIV, PIV, Gap flow, Pivoted cylinders, Mechanically coupled

1. Introduction

Arrays of cylinders are frequently seen in engineering structures, in different configurations and different Reynolds numbers. Typical examples include marine risers, offshore structures, a group of chimney stacks, heat exchanger tubes, bridge piers and cables, etc.

\textsuperscript{∗}Corresponding author. Tel.: +81 45 339 4087; fax: +81 45 339 4099.
Email address: nishi-yoshiki-rg@ynu.ac.jp (Yoshiki Nishi)
While the simplest case of flow-induced vibration (FIV) is a single circular cylinder oscillating in cross-flow, flow structure around a cylinder undergoes substantial change when it is a member of a group of cylinders in close proximity.

Zdravkovich (1988) provided an extensive study on interference-induced oscillations in flow past two circular cylinders in various configurations and classified the flow regime to three categories of flow interference; If the cylinders are far apart, the flow around either of them is similar to that of an isolated cylinder. If the cylinders are close or the second cylinder is adjacent to or within the wake of the first cylinder, the interference between the two can be one of the three types: proximity interference, wake interference, and proximity and wake interference. The origin of oscillations are also classified as jet-switch, gap-flow-switch, wake-displacement and wake-galloping. The jet-switch mechanism happens when the fluid flow passes through two bluff bodies located side-by-side. Gap-flow-switch is observed in tandem and staggered configurations where the fluid flow passes through the gap between two bluff bodies in close proximity and generates an alternating pressure that tends to close the gap and keep the bluff bodies on the center line. The wake-displacement mechanism is similar to gap-flow-switch. However, the gap-flow-switch entirely disrupts the vortex shedding and distorts the near-wake of the upstream cylinder while wake-displacement only displaces the upstream wake. The wake-displacement mechanism is usually triggered by the vortex shedding excitation while the wake-galloping mechanism characterized by extremely large amplitudes, maintained by the variation of forces exerted on the cylinder by its displacement. It was also reported that the hysteretic effect of the gap-flow-switch maintained a large amplitude oscillation when the downstream cylinder was elastically mounted with only a transverse degree of freedom.

Bokaian and Geoola (1984) also studied a case where the upstream cylinder is fixed and the downstream one is free to vibrate in the cross-flow direction for different spacing ratios and Reynolds numbers. It was found that depending on the spacing ratio, the downstream cylinder exhibited vortex excitation, galloping, or a combined VE and galloping, or separated VE and galloping. They also reported that there is a hydrodynamic restoring force in the gap between the cylinders that is acting to return the cylinder to the original position.

Although excitation mechanisms are addressed in many studies, little is known about restoring mechanism of the gap-flow-switching vibration (GSIV). Borazjani and Sotiropoulos (2009) reported that a significant portion of the incoming flow is able to pass through the gap between the two cylinders and the gap-flow mechanism starts to dominate the vortex excitation dynamics. The gap flow switching induces pressure gradients between the cylinders that results in a large oscillatory force in phase with the vortex shedding and lead to the experimentally observed larger vibration amplitudes.

Recently, to improve the energy conversion efficiency of FIV of a single circular cylinder in cross flow, a "drag assisted converter" is studied numerically by Sung et al. (2015) and tested experimentally by Arionfard and Nishi (2017). The main idea is to take advantage of the drag force by pivoting the cylinder eccentrically. While the lift force induced by vortexes starts the vibration, the drag force assists the motion depending on the pivot location and the Reynolds number.

In contrast to gap flow force, the drag force resists the restoring force in a single rota-
motionally vibrating cylinder. Our interest is to investigate the interaction of both GSIV and drag-assist force in one system. In order to have a drag-assisted vibration, it is necessary to have a rotationally oscillating cylinder. On the other hand, the most basic method of generating a GSIV is by having two circular cylinders in proximity range. Therefore, by combining these two configurations we proposed two mechanically coupled cylinders that are free to vibrate rotationally around a pivot point.

Needless to say, it is necessary to understand the vibration mechanism and vorticity dynamics in such a system, especially from the point of view of energy generation, where enhancement is the key to improve the energy conversion efficiency. Moreover, based on the idea of harnessing the energy of FIV, coupling group of energy converters as an energy farm is inevitable. Due to the physical connection between the cylinders in such a system, the vibration mechanism may be galloping, vortex excitation (VE), WIV, GSIV or a combination of them.

To the author’s best knowledge, very few publications can be found on flow around mechanically coupled cylinders. Brika and Laneville (1997) for example, reported a substantial change of the flow around the cylinder when it is mechanically coupled to a cylinder in close proximity. Even in its practical use in suspension bridges and transmission line bundles, the mechanical coupling between the cylinders is rarely considered in laboratory tests. King and Johns (1976), Hiroshi Maeda, Yoshinobu Kubo, Kusuo Kato (1997) and Cui et al. (2014) are among those few who have examined this effect. While the first two researches investigate two elastic cylinders and side by side configuration, Hiroshi Maeda, Yoshinobu Kubo, Kusuo Kato (1997) research is probably closest research to our proposed configuration. However, the drag force has no effect on the transitional vibration mechanism in their study since there is no rotation in the system.

Although the resultant forces, power and efficiency of a single pivoted cylinder is investigated in our previous study (Arionfard and Nishi, 2017), extensive research on the vibration mechanism and vorticity dynamics is necessary to understand the interaction of the flow with a group of rotationally moving cylinders. Given two mechanically coupled cylinders, it is of fundamental interest to understand: (i) how the flow-induced vibration varies with the gap between the two cylinders and the center of gravity location (ii) the resultant forces on the cylinders in different configurations and (iii) The vortex pattern characteristics in different configurations.

The present paper is focused on the first two questions while the last question is covered in a separate study. The outline of this paper is organized as follows. The experimental setup and measurement methods used in this study are explained in Section 2. The results are presented in Section 3 including a discussion about the vibration mechanisms associated to each configuration separately, followed by an overall view of all studied cases. Finally, we make conclusive remarks in Section 4.
2. Experimental setup and measurement methods

2.1. Water channel and test model

Model tests were conducted in a recirculating free surface water channel. The channel length is 1 meter with a test section dimensions of 0.3 m wide by 0.3 m deep. The channel floor and the two side walls of the test section were made of transparent acrylic to allow visual observations and particle image velocimetry (PIV) measurements. The flow rate was controlled by using a centrifugal pump fitted with a variable speed controller, so the flow velocity in the test section could be adjusted to any value between 0.024 and 0.84 m s$^{-1}$.

Two acrylic circular cylinders with a diameter of $D=0.03$ m and length of 0.24 m were used in the experiments resulting in a blockage ratio of 10%. However, on certain rotational displacements, the maximum projected area of both cylinders would result in a blockage ratio of 20%. According to Assi et al. (2010), although the maximum amplitude of oscillation decreases slightly for higher blockage ratios, the results remain qualitatively the same, meaning that the hydrodynamic mechanism will not change, based on the VIV tests on a single circular cylinder in a water channel with three ratios of blockage: 11.3, 13.6 and 17%.

It was decided not to install end plate on the cylinders, firstly, in order to decrease the fluid damping on the system in motion and secondly, to have the gap between the cylinders as small as possible. The free-stream turbulence level of the water channel on the centerline is measured between 4.3 and 10.2% based on the PIV analysis. During the experiments, the water depth is measured by an ultrasonic sensor and maintained around 0.23 m to keep the wet length of the cylinders around $L_{\text{wet}} = 0.18$, resulting in an aspect ratio ($AR$) of around 6. There is a 0.05 m gap between the bottom of the water channel and the end of cylinders. The top end of both cylinders are fixed on a connector arm by using three screws, allowing us to slide the cylinders along the connector arm and adjust the position of the cylinder and, as a result, change the center of gravity, gap between the two cylinder and the moment of inertia ($I$) of the system.

The connector arm is pivoted outside of the water by using a rigid shaft and two ball bearings, enabling rotation around the Z-axis, where X is the stream-wise coordinate, Y is the cross-stream coordinate and Z is the vertical coordinate. Different parts of the apparatus are shown in Fig.1. The configuration of the cylinders is defined by the gap ratio between the two cylinders ($G=\text{Gap}/D$) and the center of gravity ratio ($CG=\text{cg}/D$) as shown in Fig.2(a). The direction agreement of the measured forces and flow structure definitions are also illustrated in Fig.2(b,c).

There are two linear springs connected to the shaft by using a spring connector arm to provide the restoration moment for the system. Since the springs orientation is slightly changing during the rotation, the equivalent torsional spring stiffness is calculated based on the Z-moment determined by the force-moment sensor during the rotation. The average of the equivalent torsional spring stiffness is then used in the calculations and analysis. The rotation of the system is limited to $\pm 90^\circ$ from the resting position due to the springs configuration and structural limits. The equation of motion for a rotationally moving system is given by:
Figure 1: 3D view and front view of the experimental setup. (1) Camera, (2) Laser, (3) Fixing screws, (4) Cyl1, (5) Cyl2, (6) Force-moment sensor, (7) Springs, (8) Bearings, (9) Spring connector arm, (10) Shaft encoder. The red, green and blue arrows in 3d axis correspond to X, Y and Z respectively.

Figure 2: The schematics of two circular cylinders: (a) Notation, (b) Shear layer, vortex designations and positive direction of the forces and momentum coefficients. (c) Gap-flow patterns.
Figure 3: The schematics of all possible configurations. The white empty circle shows the pivot point and the filled black circle shows the location cylinders are fixed to the connector arm. The estimate range of movement for circles are also shown with faint-colored circles in each configuration.

\[ I_t \ddot{\theta} + C \dot{\theta} + K \theta = M \]  

Where \( I_t \) is the total moment of inertia of the rotating bodies, \( C \) is the damping coefficient, \( K \) is the equivalent torsional spring stiffness and \( M \) represent the total external moment around the Z-axis. It is known that the dynamic response of a cylinder is extremely sensitive to the structural characteristics of the system; therefore, extra care was taken to determine the precise value of natural frequency, the moment of inertia and damping of the structure. The base moment of inertia of all oscillating parts excluding the cylinders (\( I_0 \)) is determined by performing a series of free decay tests in the air and checked with the CAD drawings of all parts. The total moment of inertia is then calculated by summing up the moment of inertia of the cylinders around the shaft and \( I_0 \) for each test. Two bearings are used on the shaft to reduce damping and to provide more rigidity in resisting drag loads for higher Reynolds numbers. By carrying out free decay tests in air, it was also possible to estimate the structural damping of the system resulting in \( C = 0.003 \text{ kg.s}^{-1} \).

It should be noted that since the cylinders are pivoted outside of the water as surface piercing bodies, the presence of free surface affects the vorticity dynamics. The wake region close to the free surface was normally observed to be disturbed by the flow during FIV. This is due to wave run-up which developed while the cylinders are moving. The elevation of the wave run-up depends on the flow velocity and affects the wake pattern behind the cylinders near the free surface. However, the free surface effects were reported to be less significant on a high aspect ratio (\( AR > 5 \)) cylinder (A Rahman, 2015). This is due to the correlation length of the vortex shedding being increased on a long cylinder compared to a short cylinder. Consequently, the resulting force acting on the cylinders over the total length of the cylinders may be higher for a high aspect ratio (Fukuoka et al., 2016). In this study the wave run-up is lower than 15% of the submerged length and aspect ratio is more than 5, therefore, the free surface effect is not significant.

All possible configurations are shown in Fig.3 including both cylinders on the downstream, both on the upstream, one cylinder on the pivot, and a cylinder on each side of the pivot point. Total of 65 test cases with different configurations are studied by positioning the cylinders in different locations along the connector arm. In each test case, the Reynolds number is increased with a minimum step of 500 and the recording started from the onset.
of vibration and stopped if either vibration is suppressed or a maximum of $Re = 21509$ is reached, to prevent structural damage. As a result, the resolution of the recorded data is different for each case. In cases with small or no vibration, a fewer number of data point is recorded. After analyzing the results, a case with a maximum amplitude is selected from each configuration regardless of the Reynolds number and is referred to as 'selected case' in this study.

2.2. Measurement methods

PIV measurement technique is utilized in order to simultaneously obtain the velocity vectors at the PIV domain as illustrated in Fig.1. The stream-wise and transverse spans of the PIV fields are $x = 13.77D$ and $y = 6.56D$ respectively, where the origin is set to the center of the shaft (pivot point). The test section was illuminated by two thin laser light sheets from two Nd:YAG laser units with a maximum power output of 2 We each at 523 nm wavelength. The laser sheets were emitted parallel to the bottom surface of the platform and adjusted at the mid-plane of the wet length ($L_{wet}$) of the cylinders. The seeding of the water was accomplished with $12 \mu m$ diameter hollow glass sphere particles having a density of 1100 kg m$^{-3}$. It was satisfied that the particle distribution in suspension was nearly kept uniform for hours since the density of both water and seeding particles were in the similar order.

The particle images were captured using a high-speed camera (Optonics–CP80–3–M) equipped with a 60 mm lens, capable of capturing images with a resolution of 1024x1024 pixels at a sampling frequency of 333 Hz (time interval of 3 ms).

The forces and moments on the shaft are measured by using a six degrees of freedom force–moment sensor (BL–AUTOTEC MICRO6/30) and used for calculating the lift, drag and moment coefficients given by the following equations:

$$C_l = \frac{F_Y}{(\frac{1}{2} \rho D L_{wet} U^2)}$$

$$C_d = \frac{F_X}{(\frac{1}{2} \rho D L_{wet} U^2)}, C_d^* = C_d - \overline{C}_d;$$

$$C_m = \frac{M_Z}{(\frac{1}{2} \rho D L_{wet} R U^2)}$$

Where $F_Y$ and $F_X$ are the forces measured along Y and X direction respectively and $M_Z$ is the measured moment around the Z-axis. The sensor measurement accuracy is rated $\pm 2\%$ for forces and moments according to the manufacturer. $\rho$ is the mass density of the water, $U$ is the water velocity measured on the mid-span of the wet-length, $\overline{C}_d$ is the average of the drag force and $R$ is the back to back distance between the two cylinders $(R = (G + 1)D)$. The rotation of the shaft is measured by using a shaft encoder, which measures angular rotation by using two optical sensors and an encoder disk. There is no physical contact between the shaft and the encoder, hence there is no damping induced by the encoder. Although the resolution of the encoder is 1320 lines/turn ($\approx 0.3^\circ$/line), the angle between
two consecutive lines is also interpolated based on the phase difference between the signals from optical sensors resulting in an ultra precise position estimation (Kim et al., 2006).

The signals from the shaft encoder and force–moment sensor are recorded by using a Keyence data logger simultaneously. To synchronize the data captured from the camera, force–moment sensor and shaft encoder an Arduino micro-controller is used to trigger the camera and Keyence data logger at the same time.

In each set of experiment, 1500 instantaneous images are captured and recorded by a frame grabber in about 4 seconds while the data logger continues to record the force–moment sensor and shaft encoder signals for 20 seconds. The experimental data acquired by PIV system was processed with PIVlab (Thielicke and Stamhuis, 2014) package and then matched with the first 4 seconds of the force–moment sensor and shaft encoder data. Before processing the images, distortion created by the 60 mm lens is corrected by first, measuring the distortions factor using a checkboard image at the same elevation as the laser sheet and then the calculated distortion factor is used to correct the PIV images by using convex optimization in Matlab (Ying. et al., 2014).

During the post-processing in PIVlab, spurious velocity vectors due to laser sheet distortions, shadows or reflections in the flow field were omitted by means of a local median–filter technique. The eliminated velocity vectors were interpolated with surrounding vectors through a bilinear least squares fit technique. Gaussian smoothing technique is used to smooth the velocity vectors for preventing any sharp altering in the velocity field. Furthermore, the uncertainty factors in PIV technique and recording other signals could contain interrogation window size, non–uniformity in the particle distribution and size, particle overlap, electronic noises, bias and random errors, etc. It can thereby be considered that the uncertainty in the velocity measurement estimated to be around 2% (Westerweel, 1993). The calculations of vorticity values from the velocity field data were performed with a built-in curl function of Matlab where the positive vorticity is defined clockwise and the negative vorticity is counterclockwise.

2.3. Spectral analysis

To study the flow structure in the wake region of cyl1 and cyl2, spectral analysis is done by using the Fast Fourier Transformation (FFT) of the vorticity and velocity field. The vorticity field is recorded at four different points as shown in Fig.4 as stars, located at the wake side of the cylinders. Each star represents a virtual probe that records the average of the vorticity inside the dotted square areas while it moves with the cylinders to keep the probe position fixed relative to the cylinder position. Meanwhile, the local maximums and minimums of vorticity are marked (by white and black circles) and recorded on each frame to represent an overall image of vortex locations and their intensity. In addition, the velocity field is recorded inside the gap (at CG) and used to calculate the intensity of gap flow and frequency of gap switching.

Throughout this study, the dominant frequencies for displacement ($f_{osc}$), gap flow intensity ($f_C$), vorticity inside the gap ($f_{GC}$) and vorticity on the wake side ($f_{GW}$) was evaluated by determining the frequency corresponding to the largest FFT amplitude in frequency domain. The same method is used for hydrodynamic forces acting on the shaft. However, to
get a more clear frequency response for drag force, only the fluctuations from the mean drag \((C_d)\) is taken into account for the FFT analysis and referred to as \(C^*_d\). The reduced velocity
\((U^* = U/Df_N)\) was calculated by employing the natural frequency in the air \((f_N = \sqrt{K/I})\);
where \(U\) is the free stream velocity, \(K\) is the torsional stiffness and \(I\) is the total moment of
inertia. To compare with vortex shedding from a single stationary cylinder, the frequency
of natural vortex shedding \((f_v = StD/U)\) is calculated by assuming Strouhal number \((St)\)
as 0.2.

### 3. Results and discussion

#### 3.1. Vibration mechanisms

Four different vibration mechanisms are identified based on the displacement amplitude
and frequency response at different configurations shown in Fig.3: Galloping corresponds to
the a-1 configuration, wake–induces vibration (WIV) corresponds to the b-1 configuration,
vortex excitation (VE) corresponds to the a-2, b-2 and c-1 configurations and gap–flow–
switching–induced vibration (GSIV) mechanism corresponds to the c-2 configuration. It
should be noted that the mechanisms are not only a function of configuration, but also may
change with Reynolds number. Therefore, there may be an overlap between the mechanisms
corresponding to a specific configuration.

#### 3.2. a-1 and a-2 configurations

Galloping is observed when both cylinders are located on the upstream side of the pivot
(Fig. 3, a-1) while the gap is zero. An example of galloping response is shown in Fig.5(a)
for \(G=0\) and \(CG=-1.55\). To study the effect of gap ratio in this region, the results for a
case with a small gap ($G=0.37$) is shown in Fig. 5(b). In this case, the upstream cylinder moved away from the pivot while the downstream cylinder kept in place, resulting in $CG=-1.73$. The effect of GSIV can be explained by comparing the frequency response of $Cd^*$ in both cases. The fluctuations of $Cd^*$ in both cases includes a low-frequency (LF) and a high-frequency (HF) peak. In each cycle of vibration, the flow can pass throw the gap two times which greatly affects the drag force. These changes in the drag force are reflected in the frequency domain as an HF peak which is almost twice as the LF value. Therefore, the HF peak in the frequency domain of $Cd^*$ represents the gap flow intensity. It should be noted, in Fig. 5(a) and where ever $G=0$ the gap is not absolutely zero during the test, because the cylinders are fixed from the top only and hydrodynamic forces may open the gap and generate a base–bleed gap flow. The HF peak in the case with zero gap is the result of a base–bleed gap flow.

![Figure 5: An example of galloping configuration compared with 'combined GSIV and galloping' case. (a) $G=0$, $CG=-1.55$; (b) $G=0.37$, $CG=-1.73$. Both cylinders are located on the upstream in both cases.](image)

Although galloping is the dominant mechanism in both cases, the Reynolds number can change the effect of gap flow on the vibration if $G > 0$. Generally, the gap flow forms in lower Reynolds while in higher Reynolds the cylinders act as an extended body without having an effective gap flow. Here, for the case with small gap, if $U^*<9$, the LF $Cd^*$ has bigger amplitude than HF, therefore, the drag drops at the end of each cycle. A gap flow starts to form at the end of each vibration cycle. This drop of the drag force at the end of each cycle is due to the gap flow and enhances the restoring force of the spring and as a result, increases the amplitude of vibration. However, for $U^*>9$, the HF $Cd^*$ has slightly larger amplitude than LF because the cylinders act as an extended body and the drag increases at
the end of each vibration cycle. The transition between these two regions of gap flow follows by a drop in vibration amplitude around $U^*=8.9$.

The maximum measured amplitude for the galloping mechanism is $9.58^\circ$ while a small gap ($G=0.37$) increases the vibration amplitude to $20.8^\circ$ for the same $U^*$. The variation of $A_{\text{max}}$ and the normalized frequency ($f^*=f_{\text{osc}}/f_N$) with $U^*$ is shown in Fig.6(a,b) for all cases susceptible to galloping. Although $A_{\text{max}}$ increases by keeping both cylinders close to the pivot point for larger gaps, the range of having vibration narrows down. According to Fig.6(b), the frequency ratios are not close to VE resonance region even for lower $U^*$. Therefore, the cases with both cylinders on the upstream and $G<0.45$ are categorized as galloping or 'combined gap flow and galloping' region.

Moving both cylinders to the downstream side of the pivot point while the gap is zero (Fig. 3,a-2) changes the vibration mechanism to VE. The variation of the maximum amplitude with $U^*$ for $CG=1.55$ and $G=0$ is illustrated in Fig.7(a) along with the frequency analysis. To investigate the effect of gap flow on this configuration a case with $CG=1.75$ and $G=0.4$ is shown in Fig.7(b) for comparison. The natural frequency ($f_N$) for both cases is close to the vortex shedding frequency of a single stationary cylinder ($f_v$).

The displacement and frequency response for all cases with both cylinders on the downstream of the pivot point with $G=0$ (Fig.7 c, d) also confirms a vortex excitation where the oscillation frequency follows the $St=0.2$ line. Similar behavior is observed by Ryan et al. (2004) in a tethered circular cylinder which resembles the a-2 configuration. This type of vortex excitation is named VE$_{fs\_t}$ as a subcategory in vortex excitation region to distinguish between the typical lock-in and Strouhal frequency lock-in. The maximum measured amplitude for VE$_{fs\_t}$ mechanism without gap flow ($G=0$) is $6.5^\circ$ while a small gap ($G=0.4$) increases the vibration amplitude to $14.7^\circ$.

3.3. b-1 and b-2 configurations

By locating one cylinder on the pivot two interesting configurations are possible. Wake-induced vibration (WIV) is observed for the configurations where one cylinder is at the
pivot point and the other cylinder is at the downstream of the pivot (Fig. 3, b-1). The mechanism of the vibration and the relation to the gap distance is similar to Bokaian and Geoola (1984) study where a cylinder is vibrating transitionally on the wake of a stationary cylinder. However, unlike the transitional motion, the drag force assists the vibration in our case if it is not large enough to suppress the vibration as explained in Arionfard and Nishi (2017). It should be noted that the cylinder at the pivot point is rotating around its central axis and may change the vortex structure and as a result the Strouhal number in the gap (Lam, 2009). Although this effect enhances the vibration (Kimura et al., 1991), it is negligible here because the maximum non-dimensional rotation speed ($\alpha = \omega D/2U$) is 0.11 and the change of the Strouhal number in this range is less than 2% (Chew et al., 1995).

The results for three cases in the WIV region with different gap ratios are shown in Fig. 8. While For $G<1.4$ the response is a typical WIV (Fig. 8 a), for $G>1.4$ two vibration modes are recognizable similar to what Bokaian and Geoola (1984) described as 'combined vortex resonance and galloping' (Fig. 8 b,c). The first mode corresponds to $VE_{fN}$ and the next mode is due to the WIV which continues in higher reduced velocities. Comparing $U^*$ range for both cases shows that the onset of vibration is lower for larger gap ratios while the maximum amplitude is higher and more consistent for small gaps with the variation of the $U^*$. Fig. 8(a) also reveals an interesting relationship between the Reynolds number and the gap flow in small gap ratios ($G<0.5$); The amplitude of drag force fluctuations corresponding to gap flow ($HF-Cd^*$) in the case with lower Reynolds is slightly higher than the high-Reynolds case.

As mentioned in Sec. 3.2, there is a transition region in the gap flow depending on
the gap ratio and the Reynolds number. In this region, the shear layer from the upstream
cylinder is neither passes by downstream/head one nor goes through the gap. The vortex
formed from the upstream cylinder splits and part of it merges with the downstream/head,
while the rest passes through the gap (shown in Fig.2(c as ‘split-gap’). Similar behavior
is reported by Carmo et al. (2008) for two circular cylinders in staggered arrangements.
Comparing the velocity field inside the gap for both cases shows that in higher Reynolds,
the gap flow structure is ‘split-gap’ while in lower Reynolds numbers the drag drop is more
significant due to an ‘entire-gap’ flow pattern. Moreover, gap flow frequency \( f_G \) switches
from LF to HF by increasing of the Reynolds number which can be used as an identifier for
the ‘split-gap’ flow.

Total of five arrangements are tested in the WIV region and the results for displacement
and frequency response are plotted in Fig.9(a, b). While the normalized frequency follows
the \( St = 0.2 \) line in low \( U^* \), there is no significant trace of a frequency associated with
\( St = 0.2 \) beyond the reduced velocity of 6.

Having the moving cylinder on the upstream side of the pivot (Fig. 3, b-2), shifts the
vibration mechanism from WIV to galloping if the gap is small \( (G<0.5) \). Two arrangements
of having one cylinder on the pivot point and another at the upstream are compared for
small gap (Fig.10, a) and large gap (Fig.10, b). When the gap is small, the stationary
cylinder located on the pivot, acts as a vortex suppressor on the wake side and decreases the
vibration amplitude for lower reduced velocities \( (U^*<8) \). However, for \( U^*>8 \) the drag force
assist the vibration and increases the displacement amplitude by reducing the restoring
moment while the system is on the extremums of the displacement. On the other hand,
a larger gap \( (G>1.4) \) lets the flow pass through and reduces the drag force as a result.
Since the stationary cylinder effect on the wake of the upstream cylinder is negligible, the
upstream cylinder experiences VE, even though both gap-switching flow and the drag force
are enhancing the vibration amplitude. The transition from the galloping to the ‘drag and
gap–switching–assisted vibration’ is recognizable by comparing the frequency and amplitude responses of all cases with one cylinder on the pivot point and another cylinder upstream as shown in Fig.10 (c,d).

3.4. c-1 and c-2 configurations

Fig.11 shows the result for the cases where one cylinder is located on each side of the pivot point while $CG=0$. Unlike the VE$_{fSt}$ region, the frequency responses deviate from the $St=0.2$ and lock into the natural frequency (VE$_{fN}$) as shown in Fig.12 (b). VE$_{fN}$ takes place when $CG=0$ and the gap is large enough ($>3.9$) that two vortex streets are formed behind each cylinder. The vortices from the upstream cylinder may pair with the vortices shed behind the downstream cylinder or pass by the downstream cylinder depending on the natural frequency ($f_N$) and the Reynolds number.

To explain the effect of gap flow in this region a case with $CG=0$ and $G=4.9$ is compared to a case with $CG=0$ and $G=2.9$ in Fig.11(b). The amplitude response is slightly higher in most reduced velocities when the gap is smaller but there is a sudden drop in amplitude response at $U^*\approx 6.3$. Although $f_N \approx f_v$, the downstream cylinder suppresses the vortex shedding because the oscillation is not in synchronization with the vortices shed from the upstream cylinder. Similar effect is observed for all configurations with $CG=0$ especially for $1.9 < G < 2.4$. Unlike the galloping mechanism, the slight enhancement of the amplitude response in the case with smaller gap here, is due to the GSIV, because the gap flow switches direction twice in each cycle and generates alternating negative pressure inside the gap. This negative pressure generates a restoring moment around the pivot point and increases the vibration amplitude subsequently.

VE$_{fN}$ also observed for cases with slightly off-center CG as shown in Fig.12 (a,b). Comparing to VE$_{fSt}$ cases, the average $A_{max}$ is higher for VE$_{fN}$ due to having two vortices shed from each cylinder in each cycle. However, the vibration frequency for VE$_{fSt}$ cases follows the $St = 0.2$ line more closely.

Figure 9: Responses for the displacement and normalized frequency for all susceptible cases to wake-galloping.

---

**Figure 9**

Responses for the displacement and normalized frequency for all susceptible cases to wake-galloping.

(a) $A_{max}$ (deg)

(b) $f_{oc}/f_N$

- $G=0, CG=0.55$
- $G=0.4, CG=0.75$
- $G=0.9, CG=1$
- $G=1.45, CG=1.22$
- $G=1.95, CG=1.47$

![Graph showing responses](image-url)
Figure 10: Responses and frequencies for cases with one cylinder on the pivot point and another cylinder on the upstream side. (a) $G = 0.4$, $CG = -0.75$, (b) $G = 1.95$, $CG = -1.47$, (c) Maximum displacement amplitude response verses the reduced velocity and (d) Normalized frequency of oscillation verses the reduced velocity.

Figure 11: Two example cases of $VE_{fN}$ mechanism: (a) $CG = 0$, $G = 4.9$ (b) $CG = 0$, $G = 2.9$. The cylinders are located on an equal distance from the pivot point.
Figure 12: $\text{VE}_fN$ response for the cases with $CG \approx 0$ and $G \geq 3.9$. (a) Displacement response. (b) Normalized frequency response.

Since the flow structure in the gap is related to the drag force, the ratio between HF and LF amplitudes of the drag fluctuations ($R_{Cd*}$) is calculated and used for determining the dominant type of the flow structure in the gap. No significant gap flow is observed for small amplitudes, therefore, only the cases with the highest amplitude are selected in each configuration, regardless of the Reynolds number. An interpolated contour plot for the variation of the $R_{Cd*}$ with $CG$ and $G$ for the selected data is shown in Fig.13. $R_{Cd*} > 1$ here, indicates a stronger HF fluctuation of $Cd*$, therefore, stronger gap flow. Two peaks are recognizable on this figure at P1($CG=1.225, G=1.45$) and P2($CG=0, G=1.9$). The range of $CG$ for cases susceptible to wake galloping is from 0.55 to 1.475 as marked in Fig.13 where peak (P1) is also located. Therefore for cases within this range, critical gap ratio for wake-induced vibration is $1 < G < 2$ with maximum gap flow intensity at $G=1.45$. The other peak point (P1), is located on the GSIV region with the maximum gap flow intensity at $G=1.9$.

A high Reynolds gap flow reduces the pressure inside the gap. This reduction in pressure acts as a restoring moment and enhances the vibration by forcing the cylinders to return to the centerline. However, a strong gap flow is not necessarily enhancive; If both cylinders are located at the same side, low-pressure zones on the head of cyl1 and the base of cyl2 generates opposite moments around the pivot and cancel each other. However, for $-0.5 < CG < 0.75$ the generated moment directions are same, which results in a dominant GSIV in this region. The second peak (P2) is associated with this range. According to Fig.13 when $G<2.9$, HF amplitude of $Cd*$ starts to increase by decrease in gap ratio and then drops suddenly for $G<0.5$. It should be noted that an increase in the Reynolds number, dramatically shifts the minimum gap required for having an entire-gap flow. Therefore, for higher Reynolds numbers, a wider gap is required to initiate an entire-gap flow regime.

Fig.14(a) compares the variation of the maximum amplitude with $U^*$ for four different gap ratios while $CG=0$ and gives a good picture of transition between $\text{VE}_{fSt}$ and GSIV. When $G=2.9$, two response modes are clearly identifiable but for $G=4.9$ the response is $\text{VE}_{fSt}$ with a very small decrease in $A_{\text{max}}$ around $U^*=6$ due to the wake interaction. However, for
$G=2.9$ this reduction in amplitude separates the responses to two modes of vortex resonance and gap switching which is a transition state between $\text{VE}_{\text{fSt}}$ and GSIV. Reducing the gap ratio to $G=1.9$ starts to increase the average vibration amplitude in both modes and the GSIV amplitude response continues to grow for higher $U^*$. When the gap reaches $G=0.9$ the average of vibration amplitude dramatically increases, especially for $U^*>7$ which is a result of a strong gap flow switching. Since the pivot point is at the center, the resultant moment of the drag force on the cylinders is zero but a small slide of the CG changes the response due to the drag force effect, depending on the configuration and the Reynolds number.

Fig. 14(b) shows the effect of sliding both cylinders towards the upstream or downstream while the gap is kept constant at $G=0.9$. When $CG=-0.5$ the drag force assist the motion as explained in Arionfard and Nishi (2017) and slightly increases the maximum amplitude by reducing restoring moment at the extremums of vibration. However, in higher Reynolds numbers, the drag-assist effect resists the gap flow effect and reduces the total moment around the pivot point and decreases the vibration amplitude subsequently. The displacement response is shown in the background in gray.

Although in GSIV the magnitude of the gap velocity increases and the drag decreases as the amplitude of vibration goes up, there is a phase lag between the displacement and drag reduction. Moreover, the gap flow velocity is also not in phase with the displacement response. The displacement, $Cd^*$ and gap flow time histories for a case with the maximum $A_{\text{max}}$ ($G = 0.9, CG = 0$) are shown in Fig. 15(a). All responses are scaled and centered for better visualization of the phase difference. Both gap flow and $Cd^*$ comprise two frequencies.
Figure 14: The variation of the maximum displacement amplitude with the reduced velocity for cases in the GSIV region. (a) Effect of gap ratio. (b) Effect of $CG$.

Figure 15: Time histories for the case with maximum $A_{\max}$ ($G=0.9$, $CG=0$). (a) displacement, $Cd^*$ and gap flow velocity response. (b) separated high frequency components for $Cd^*$ and gap flow velocity.
as mentioned before. Therefore, the frequencies and phases for each response are found by fitting the data on two terms of a Fourier series including a phase constant. The low-frequency responses are not associated with the gap flow and drag reduction. Hence, only the high frequency components of the responses are plotted in Fig.15(b) along with the displacement response. Although the minimum of the high-frequency \( Cd^* \) (HF\(-Cd^*\)) is close to the extremums of the displacement response as expected, maximum of gap flow is about 90° out of phase from the displacement response. Moreover, HF\(-Cd^*\) is also −30° out of phase compared to the displacement response. Similar pattern is observed for all cases with GSIV which is a result of passing vortexes through the gap, especially for \( G>0.4 \). While passing a vortex through the gap reduces the drag force, the strong vorticity inside the gap drains energy by redirecting the flow and as a result reducing the velocity magnitude of the gap flow.

3.5. The effect of the configuration and Reynolds number.

To have an overall image of the relationship between the Reynolds number, configuration and displacement amplitude, the selected cases are divided into ten groups based on the Reynolds number. For each case, the variation between the \( G, CG \) and \( A_{\text{max}} \) is plotted as a linearly interpolated contour plot. The results for all ten groups is then illustrated in Fig.16. A lighter color is associated with a higher \( A_{\text{max}} \) as shown in the color map. It is possible to identify the transition between the vibration mechanisms by following the white spot from the highest Reynolds group (\( 21020 \leq Re \leq 2395 \)) to the lowest one. As the Reynolds number decreases, the white spot shifts from \( CG = 0 \) where GSIV is the dominant mechanism to the left side (\( CG < 0 \)) where the drag force assists the vibration. While \( A_{\text{max}} \) is still higher around \( CG = 0 \) for \( 17237 \leq Re \leq 19359 \), the gap flow strength starts to reduce and as a result the white spot separates to two spots on galloping and drag-assisted GSIV (\( 13877 \leq Re \leq 15552 \)). As the Reynolds number reduces, the white spot starts to spread as moves upward and right (\( 8885 \leq Re \leq 10537 \)) in which VE\(_{fSt}\) and WIV are dominant due to having a larger gap, low Reynolds number and a weaker gap flow.

4. Conclusions

The vibration mechanism of two mechanically coupled cylinders oscillating rotationally is investigated by water channel tests. Four regions of vibration are identified by varying the Reynolds number and the location of the cylinders. Galloping is observed when both cylinders are located on the upstream side of the pivot point and the gap between them is zero. However, a small gap (0.37\( D \)) between the cylinders initiates a gap flow that enhances the vibration amplitude by reducing the drag force around the extremums of the displacement and as a result reduces the resistance on the restoring moment. Locating both cylinders on the downstream side of the pivot point changes the vibration mechanism to VE\(_{fSt}\). In this region the frequency responses are close to the vortex shedding frequency of a single circular cylinder oscillating in cross-flow direction (\( f_v \)). The VE\(_{fN}\) occurs if a cylinder is located on each side of the pivot at equal distances. Although frequency response in this region is also close to \( f_v \), the vibration is more sensitive to \( f_N \). Since the cylinders
Figure 16: An overall image of configuration effect on the displacement amplitude for ten groups of Reynolds numbers.
are mechanically coupled, the vibration may be suppressed if the motion is not in sync with
the vortex shedding from the upstream cylinder.

If one cylinder is located on the pivot point the system is susceptible to wake-induced
vibration; specifically if the other cylinder is on the wake side (downstream). The cases
where gap is zero, follow a typical wake-galloping response in this region. However, hav-
ing a small gap between the cylinders shifts the vibration to 'combined vortex resonance
and galloping'. Although gap flow affects all cases with \( G > 0 \), GSIV is dominant for -
\( 0.5 < CG < 0.75 \) especially for \( G < 2.9 \). This is found by comparing HF and LF amplitudes of
\( Cd^* \). Decomposing the LF components of the \( Cd^* \) and gap flow velocity and comparing
it to the displacement respond shows a \(-30^\circ\) and \(90^\circ\) phase shift from the displacement
response respectively due to passing vortices through the gap. The maximum amplitude
occurs in GSIV region in high Reynolds(\(21020 \leq Re \leq 2395\)) and shifts to galloping region
for \(13877 \leq Re \leq 15552\). Although there is no significant change in the maximum displace-
ment amplitude for \( Re < 12189 \), the dominant vibration mechanisms are \( VE_{fs} \) and \( WIV \n\)

5. Acknowledgment

This study was financially supported by the Grant-in-Aid for Scientific Research 15H04211.
The authors also acknowledge the contribution of Kengo Fukuda in setting up the experi-
ment and conducting the tests.

References

thesis, The University of Western Australia.
Arionfard, H., Nishi, Y., 2017. Experimental investigation of a drag assisted vortex-induced vibration energy
URL http://dx.doi.org/10.1016/j.jfluidstructs.2016.10.002
URL papers3://publication/uuid/349C339F-A934-4BFA-B78E-975313BE982
Borazjani, I., Sotropoulos, F., jan 2009. Vortex-induced vibrations of two cylinders in tandem arrangement
pmcsearch(&)rendertype=abstract
Bria, D., Laneville, A., 1997. Vortex-induced oscillations of two flexible circular cylinders coupled mechan-
URL http://linkinghub.elsevier.com/retrieve/pii/S0167610597001621
URL http://www.journals.cambridge.org/abstract_S00221296003417

URL http://dx.doi.org/10.1016/j.jfluidstructs.2013.11.007


URL http://ieeexplore.ieee.org/lpdocs/epic03/wrapper.htm?arnumber=1620650


